PROBLEMS

Introductory Problems

1-1* Calculate the mass m of a body that weighs 600 lb at the surface of the earth.

1-2* Calculate the weight W of a body at the surface of the earth if it has a mass m of 675 kg.

1-3 Calculate the gravitational constant g, in U.S. Customary units, for a location on the surface of the moon.

1-4 Calculate the gravitational constant g, in SI units, for a location on the surface of the sun.

1-5* The planet Venus has a diameter of 7700 mi and a mass of $3.34(10^{23})$ slug. Determine the gravitational acceleration at the surface of the planet.

1-6* The gravitational acceleration at the surface of Mars is $3.73\ \text{m/s}^2$, and the mass of Mars is $6.39(10^{23})$ kg. Determine the radius of Mars.

1-7 Determine the gravitational force, in kips, exerted by the sun on the earth.

1-8 Determine the gravitational force, in kilonewtons, exerted by the earth on the moon.

1-9* The equatorial radius of the earth is $2.0925(10^7)$ ft, and the polar radius is $2.0856(10^7)$ ft. Determine the gravitational acceleration g at the two locations.

1-10* Two spherical bodies have masses of 60 kg and 80 kg, respectively. Determine the gravitational force of attraction between the spheres if the distance from center to center is 600 mm.

1-11 Two solid spherical bodies have 12 in. and 10 in. diameters and are made of a material that weighs $0.284\ \text{lb/in.}^3$. Determine the gravitational force of attraction between the two spheres when they are touching each other.

1-12 A satellite is placed in orbit $1.6(10^6)$ m above the surface of the moon. If the mass of the satellite is $3.0(10^4)$ kg, determine the gravitational force exerted on the satellite by the moon.

Intermediate Problems

1-13* Determine the weight W of a satellite when it is in orbit 8500 mi above the surface of the earth if the satellite weighs 7600 lb at the surface.

1-14* Determine the weight W of a satellite when it is in orbit $20.2(10^6)$ m above the surface of the earth if the satellite weighs 8450 N at the surface.

1-15 If a woman weighs 135 lb when standing on the surface of the earth, how much would she weigh when standing on the surface of the moon?

1-16 Determine the weight W of a body that has a mass of 1000 kg

a. At the surface of the earth.
b. At the top of Mt. McKinley (6193 m above sea level).
c. In a satellite at an altitude of 250 km.

1-17* If a man weighs 210 lb at sea level, determine the weight W of the man

a. At the top of Mt. Everest (29,028 ft above sea level).
b. In a satellite at an altitude of 200 mi.

1-18 A space traveler weighs 800 N on earth. A planet having a mass of $5(10^{25})$ kg and a diameter of $30(10^6)$ m orbits a distant star. Determine the weight W of the traveler on the surface of this planet.

Challenging Problems

1-19* The planet Jupiter has a mass of $1.302(10^{26})$ slug and a visible diameter (top of cloud layer) of 88,700 mi. Determine the gravitational acceleration g

a. At a point 100,000 miles above the top of the clouds.
b. At the top of the cloud layers.

1-20* The planet Saturn has a mass of $5.67(10^{26})$ kg and a visible diameter (top of cloud layer) of $12.00(10^7)$ m. The weight W of a planetary probe on earth is 4.50 kN. Determine

a. The weight of the probe when it is $6(10^8)$ m above the top of the clouds.
b. The weight of the probe as it begins its penetration of the cloud layers.

1-21 The first U.S. satellite, *Explorer I*, had a mass of approximately 1 slug. Determine the force exerted on the satellite by the earth at the low and high points of its orbit, which were 175 mi and 2200 mi, respectively, above the surface of the earth.

1-22 A neutron star has a mass of $2(10^{30})$ kg and a diameter of $10(10^3)$ m. Determine the gravitational force of attraction on a 10-kg space probe

a. When it is 10^6 m from the center of the star.
b. At the instant of impact with the surface of the star.

1-23* At what distance from the surface of the earth, in miles, is the weight of a body equal to one-half of its weight on the earth's surface?

1-24 At what distance, in kilometers, from the surface of the earth on a line from center to center would the gravitational force of the earth on a body be exactly balanced by the gravitational force of the moon on the body?

PROBLEMS

Introductory Problems

1-25* Determine the weight W, in U.S. Customary units, of an 85-kg steel bar under standard conditions (sea level at a latitude of 45 degrees).

1-26* Determine the mass m, in SI units, for a 600-lb steel beam under standard conditions (sea level at a latitude of 45 degrees).

1-27 The velocity of light in space is approximately 186,000 mi/s. The velocity of light in units of kilometers per hour is?

1-28 Using the fact that 1 m = 39.37 in., convert 5 m^3 of concrete to units of cubic yards of concrete.

1-29* Using the fact that 1 in. = 25.40 mm, convert a speed of 75 mi/h to units of meters per second.

1-30* The fuel consumption of an automobile is 14 km/L. The fuel consumption, in miles per gallon, is?

1-31 An automobile has a 350-$in.^3$ engine displacement. The engine displacement, in liters, is?

1-32 How many barrels of oil are contained in 100 kL of oil? One barrel (petroleum) equals 42.0 gal.

1-33* Express a speed of 20 nm/h (1 nautical mile = 6076 ft) in units of kilometers per minute.

1-34* One acre equals 43,560 ft^2. One hectare equals 10^4 m^2. Determine the number of acres in 500 hectares.

1-35 Verify the conversion factors listed in Table 1-6 for converting the following quantities from U.S. Customary units to SI units by using the values listed for length as defined values:

a. Velocity b. Acceleration

1-36 Verify the conversion factors listed in Table 1-6 for converting the following quantities from SI units to U.S. Customary units by using the values listed for length as defined values:

a. Area b. Volume

Use 1 gal = 231 $in.^3$ and 1 L = 0.001 m^3.

Intermediate Problems

1-37 Verify the conversion factors listed in Table 1-6 for converting the following quantities from U.S. Customary units to SI units by using the values listed for length and force as defined values:

a. Mass b. Distributed load

1-38 Verify the conversion factors listed in Table 1-6 for converting the following quantities from SI units to U.S. Customary units by using the values listed for length and mass as defined values:

a. Pressure or stress b. Bending moment or torque

1-39 One acre equals 43,560 ft^2. One gallon equals 231 $in.^3$. Determine the number of liters of water in 2500 acre · ft of water.

1-40 The viscosity of crude oil under conditions of standard temperature and pressure is $7.13(10^{-3})$ N · s/m^2. The viscosity of crude oil in U.S. Customary units (lb · s/ft^2) is?

1-41* The air pressure in an automobile tire is 35 psi. Express the pressure in appropriate SI units (kPa) by using the values listed in Table 1-6 for length and force as defined values.

1-42 The stress in a steel bar is 150 MPa. Express the stress in appropriate U.S. Customary units (ksi) by using the values listed in Table 1-6 for length and force as defined values.

Challenging Problems

1-43* Express the density, in SI units (kg/m^3), of a specimen of material that has a specific weight of 0.284 lb/$in.^3$.

1-44* Express the specific weight, in U.S. Customary units (lb/$in.^3$), of a specimen of material that has a density of 4500 kg/m^3.

1-45 By definition, 1 hp = 33,000 ft · lb/min and 1 W = 1 N · m/s. Verify the conversion factors listed in Table 1-6 for converting power from U.S. Customary units to SI units by using the values listed for length and force as defined values.

1-46 The specific heat of air under standard atmospheric pressure, in SI units, is 1003 N · m/kg · K. The specific heat of air under standard atmospheric pressure, in U.S. Customary units (ft · lb/slug · °R), is?

EXAMPLE PROBLEM 1-6

Determine the dimensions of I, R, w, M, and C in the dimensionally homogeneous equation

$$EIy = Rx^3 - P(x - a)^3 - wx^4 + Mx^2 + C$$

in which x and y are lengths, P is a force, and E is a force per unit area.

SOLUTION

The equation can be written dimensionally as

$$\frac{F}{L^2}(I)(L) = R(L^3) - F(L - a)^3 - w(L^4) + M(L^2) + C$$

For this equation to be dimensionally homogeneous a must be a length; hence, all terms must have the dimensions FL^3. Thus,

$$(I)\frac{F}{L} = (R)L^3 = (w)L^4 = (M)L^2 = C = FL^3$$

Every term in a dimensionally homogeneous equation must have the same dimension.

The dimensions for each of the unknown quantities are obtained as follows:

$$I = \frac{L}{F}(FL^3) = L^4 \qquad \text{Ans.}$$

$$R = \frac{1}{L^3}(FL^3) = F \qquad \text{Ans.}$$

$$w = \frac{1}{L^4}(FL^3) = \frac{F}{L} \qquad \text{Ans.}$$

$$M = \frac{1}{L^2}(FL^3) = FL \qquad \text{Ans.}$$

$$C = FL^3 \qquad \text{Ans.}$$

PROBLEMS

Introductory Problems

1-47* Newton's law of gravitation can be expressed in equation form as

$$F = G\frac{m_1 m_2}{r^2}$$

If F is a force, m_1 and m_2 are masses, and r is a distance, determine the dimensions of G.

1-48* The elongation of a bar of uniform cross section subjected to an axial force is given by the equation

$$\delta = \frac{PL}{AE}$$

What are the dimensions of E if δ and L are lengths, P is a force, and A is an area?

1-49 An important parameter in certain types of fluid flow problems when a free surface is present is the Froude number (Fr), which can be expressed in equation form as

$$\text{Fr} = \left[\frac{\rho v^2}{Lw}\right]^{1/2}$$

where ρ is the density of the fluid, v is a velocity, L is a length, and w is the specific weight of the fluid. Show that the Froude number is dimensionless.

1-50 An important parameter in fluid flow problems involving thin films is the Weber number (We) which can be expressed in equation form as

$$\text{We} = \frac{\rho v^2 L}{\sigma}$$

where ρ is the density of the fluid, v is a velocity, L is a length, and σ is the surface tension of the fluid. If the Weber number is dimensionless, what are the dimensions of the surface tension σ?

1-51* The period of oscillation of a simple pendulum is given by the equation

$$T = k(L/g)^{1/2}$$

where T is in seconds, L is in feet, g is the acceleration due to gravity, and k is a constant. What are the dimensions of k for dimensional homogeneity?

1-52* In the equation

$$y = y_0 + vt + \frac{1}{2}at^2$$

y and y_0 are distances, v is a velocity, a is an acceleration, and t is time. Is the equation dimensionally homogeneous?

1-53 The modulus k of a coil spring (force required to stretch the spring a unit distance) can be expressed in equation form as

$$k = \frac{Gr^4}{4R^3n}$$

in which r and R are lengths and n is a dimensionless number. Determine the dimensions of G (a property of the spring material).

1-54 In the dimensionally homogeneous equation

$$U = Fd - \frac{Wv^2}{2g}$$

F is a force, W is a force, d is a length, and v is a linear velocity. Determine the dimensions of U and g.

Intermediate Problems

1-55* In the dimensionally homogeneous equation

$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$

σ is a stress, A is an area, M is a moment of a force, and c is a length. Determine the dimensions of P and I.

1-56* In the dimensionally homogeneous equation

$$Pd = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

d is a length, m is a mass, v is a linear velocity, and ω is an angular velocity. Determine the dimensions of P and I.

1-57 In the dimensionally homogeneous equation

$$\tau = \frac{Tr}{J} + \frac{VQ}{Ib}$$

τ is a stress, T is a torque, V is a force, r and b are lengths, and I is a second moment of an area. Determine the dimensions of J and Q.

1-58 In the dimensionally homogeneous equation

$$\tau = \frac{P}{A} + \frac{Tr}{J}$$

τ is a stress, A is an area, T is a torque, and r is a length. Determine the dimensions of P and J.

Challenging Problems

1-59* The equation $x = Ae^{-t/b}\sin(at + \alpha)$ is dimensionally homogeneous. If A is a length and t is time, determine the dimensions of x, a, b, and α.

1-60* In the dimensionally homogeneous equation $w = x^3 + ax^2 + bx + a^2b/x$, if x is a length, what are the dimensions of a, b, and w?

1-61 Determine the dimensions of a, b, c, and y in the dimensionally homogeneous equation

$$y = Ae^{-bt}\cos\left[\sqrt{1 - a^2}\,bt + c\right]$$

in which A is a length and t is time.

1-62 Determine the dimensions of c, ω, k, and P in the differential equation

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = P\cos\omega t$$

in which m is mass, x is length, and t is time.

1-6 METHOD OF PROBLEM SOLVING

The principles of mechanics are few and relatively simple; however, the applications are infinite in their number, variety, and complexity. Success in engineering mechanics depends to a large degree on a well-disciplined method of problem solving. Experience has shown that the development of good problem-solving methods and skills results from solving a large variety of problems. Professional problem solving consists

EXAMPLE PROBLEM 1-7

Round off the number 12,345 to two, three, and four significant figures. Find the percent difference between the rounded-off numbers and the original number by using the original number as the reference.

SOLUTION

Rounding off the number 12,345 to two, three, and four significant figures yields 12,000, 12,300, and 12,350. The percent difference for each of these numbers is

$$\%D = \frac{A - B}{B}(100)$$

For 12,000,

$$\%D = \frac{12{,}000 - 12{,}345}{12{,}345}(100) = -2.79\% \qquad \text{Ans.}$$

For 12,300,

$$\%D = \frac{12{,}300 - 12{,}345}{12{,}345}(100) = -0.36\% \qquad \text{Ans.}$$

For 12,350,

$$\%D = \frac{12{,}350 - 12{,}345}{12{,}345}(100) = +0.041\% \qquad \text{Ans.}$$

The minus signs associated with the above percent differences indicate that the rounded-off numbers are smaller than the reference number. Similarly, a positive percent difference indicates that the rounded-off number is larger than the reference number.

PROBLEMS

Round off the numbers in the following problems to two significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

1-63*	a. 0.0153	b. 0.0347	c. 0.0566
1-64*	a. 0.8374	b. 0.4729	c. 0.6644
1-65	a. 1.8394	b. 3.4629	c. 6.7523
1-66	a. 3.6544	b. 7.5638	c. 8.9223

Round off the numbers in the following problems to three significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

1-67*	a. 26.394	b. 74.829	c. 55.336
1-68*	a. 374.93	b. 826.48	c. 349.33
1-69	a. 6471.9	b. 3628.7	c. 7738.2
1-70	a. 8521.4	b. 6748.3	c. 9378.7

Round off the numbers in the following problems to four significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

1-71*	a. 63,746.2	b. 27,382.6	c. 55,129.9
1-72*	a. 937,284	b. 274,918	c. 339,872
1-73	a. 918,273	b. 284,739	c. 342,691
1-74	a. 624,373	b. 785,239	c. 936,491

Computer Problems

C1-75 A common practice in rounding answers is to report numbers whose leading digit is 1 to an accuracy of 4 significant figures and all other numbers to an accuracy of 3 significant figures. Although this practice probably started with the accuracy with which slide rules could be read, it also reflects the fact that an accuracy of greater than 0.2 percent is seldom possible. This project will examine the error introduced by this and some other rounding schemes. For each of the rounding schemes below,

1. Generate 20,000 random numbers between 1 and 10.
2. Round each number to the specified number of significant figures. (Note that 3 significant figures is equivalent to 2 decimal places, 4 significant figures is equivalent to 3 decimal places, etc., since all numbers are between 1 and 10.)
3. Calculate the percent relative error for each number.

`PercentRelError =`

$$\left|\frac{\texttt{Number} - \texttt{RoundNumber}}{\texttt{Number}}\right| \texttt{*100}$$

4. Plot `PercentRelError` versus `Number`.
5. Comment on the maximum round-off error and the distribution of round-off error.

a. Round all numbers to an accuracy of 3 significant figures.
b. Round numbers less than 2 to an accuracy of 4 significant figures and numbers greater than 2 to an accuracy of 3 significant figures.
c. Round numbers less than 3 to an accuracy of 4 significant figures and numbers greater than 3 to an accuracy of 3 significant figures.
d. Round numbers less than 5 to an accuracy of 4 significant figures and numbers greater than 5 to an accuracy of 3 significant figures.

C1-76 When engineers deal with angles, they are usually more interested in the sine or cosine of the angle than they are with the angle itself. Since

$$\sin 5^\circ = \cos 85^\circ = \sin 175^\circ = \sin 1085^\circ = \cdots = 0.08176$$

the rounding of angles requires a different scheme than that described in Problem C1-75. That is, angles should be rounded to a specified number of decimal places rather than a specified number of significant figures. This project will examine the error introduced by rounding angles to various numbers of decimal places. For each of the cases below,

1. Generate 20,000 random angles between 1° and 89°. (Use a random number generator that produces decimal numbers and not just integers.) Calculate the sine and cosine of each angle.
2. Round each angle to the specified number of decimal places and calculate the sine and cosine of the rounded angle.
3. Calculate the percent relative error for each angle.

`PercentRelError =`

$$\left|\frac{\texttt{sin(Angle)} - \texttt{sin(RoundAngle)}}{\texttt{sin(Angle)}}\right| \texttt{*100}$$

or

`PercentRelError =`

$$\left|\frac{\texttt{cos(Angle)} - \texttt{cos(RoundAngle)}}{\texttt{cos(Angle)}}\right| \texttt{*100}$$

4. Plot `PercentRelError` versus `Angle`.
5. Comment on the maximum round-off error and the distribution of round-off error.

a. Round all angles to an accuracy of 1 decimal place.
b. Round all angles to an accuracy of 2 decimal places.
c. Round angles less than 10° to an accuracy of 3 decimal places and angles greater than 10° to an accuracy of 2 decimal places.

C1-77 When two numbers are added or multiplied together, the result is always less accurate than the original numbers. This project will examine the error introduced by rounding two numbers before they are multiplied together.

a. Generate 80 random numbers between 4.51 and 5.49 (that is, 5 ± 0.1*RND). If any pair of these numbers are rounded to the nearest integer (5) and then multiplied together, the result will be 25. How does this result compare with the correct product obtained by multiplying the original two numbers together? Is the result accurate to the nearest integer? Is the result accurate to less than 10 percent?
b. Repeat part *a* for numbers between 49.51 and 50.49 (50 ± 0.01*RND). Is the result accurate to the nearest integer? Is the result accurate to less than 1 percent?
c. Generate 20,000 random integers between 1 and 49. For each integer N, generate two random numbers that will round to that integer

`N1 = N ± 0.5*RND`   `N2 = N ± 0.5*RND`

Plot the percent relative difference in the products

$$\texttt{ProdDiff} = \left|\frac{\texttt{N1*N2} - \texttt{N*N}}{\texttt{N*N}}\right| \texttt{*100}$$

versus N. Compare this with the percent relative difference in the original numbers

$$\texttt{NumDiff} = \left|\frac{\texttt{N1} - \texttt{N}}{\texttt{N}}\right| \texttt{*100}$$

REVIEW PROBLEMS

1-78* The planet Neptune has a mass of $1.03\,(10^{26})$ kg and a visible diameter (top of cloud layers) of $4.86 \times (10^7)$ m. Determine the gravitational acceleration g at the top of the cloud layers.

1-79* The weight of the first Russian satellite, *Sputnik I*, on the surface of the earth was 184 lb. Determine the force exerted on the satellite by the earth at the low and high points of its orbit which were 149 mi and 597 mi, respectively, above the surface of the earth.

1-80 The planet Jupiter has a mass of $1.90(10^{27})$ kg and a radius of $7.14(10^7)$ m. Determine the force of attraction between the earth and Jupiter when the minimum distance between the two planets is $6(10^{11})$ m.

1-81 On the surface of the earth the weight of a body is 200 lb. At what distance from the center of the earth would the weight of the body be (a) 100 lb? (b) 50 lb?

1-82* At what distance from the center of the earth would the force of attraction between two spheres 1 m in diameter in contact equal the force of attraction of the earth on one of the spheres? The mass of each sphere is 250 kg.

1-83* The weight of a satellite on the surface of the earth prior to launch is 250 lb. When the satellite is in orbit 5000 miles from the surface of the earth, determine the force of attraction between the earth and the satellite.

1-84 A fluid has a dynamic viscosity of $1.2(10^{-3})\ \mathrm{N \cdot s/m^2}$. Express its dynamic viscosity in U.S. Customary units ($\mathrm{lb \cdot s/ft^2}$).

1-85 The stress equation for eccentric loading of a short column is

$$\sigma = -\frac{P}{A} - \frac{Pey}{I}$$

If P is a force, A is an area, and e and y are lengths, what are the dimensions of stress σ and second moment of area I?

1-86* Determine the dimension of c in the dimensionally homogeneous equation

$$v = \frac{mg}{c}\left[1 - e^{-ct/m}\right]$$

in which v is a velocity, m is a mass, t is time, and g is the gravitational acceleration.

1-87* In the dimensionally homogeneous equation

$$R = cv + ag$$

R is a force, v is a velocity, and g is an acceleration. Determine the dimensions of a and c.

1-88 When a body moves through a fluid it experiences a resistance to its motion which can be represented by the equation $F = \frac{1}{2}C_D\rho V^2 A$ where F is a force, ρ is the density of the fluid, V is the velocity of the body relative to the fluid, and A is the cross-sectional area of the body. Show that the drag coefficient C_D is dimensionless.

1-89 Develop an expression for the change in gravitational acceleration Δg between the surface of the earth and a height h when $h << R_e$.